Developing number sense: What can other cultures tell us?

*Teaching Children Mathematics*; Reston; Feb 2001; Claudia Zaslavsky;

<table>
<thead>
<tr>
<th>Volume:</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>Issue:</td>
<td>6</td>
</tr>
<tr>
<td>Start Page:</td>
<td>312</td>
</tr>
<tr>
<td>ISSN:</td>
<td>10735836</td>
</tr>
</tbody>
</table>

Full Text:

Copyright National Council of Teachers of Mathematics Feb 2001

Linda Lai's third- and fourth-grade mathematics students in the Edith Bowen Lab School of Utah State University are studying a variety of counting systems used by people around the world and throughout time. Activity and voices fill the room as students use books, reference materials, and even the Internet to conduct their research. As the students work, many show surprise when they learn about and experience the many ways that people have counted and recorded numbers. From finger counting to number words that relate to our fingers and toes; to concrete materials, such as sticks, strings, and pebbles; and, finally, to the numerals invented by various societies, these students are enhancing their number sense by learning about the systems of other cultures. Additionally, the students are gaining a cultural appreciation for the diverse ways that we all use to count.

Across the country Bianka Crespo's fifth-grade class at Salome Urena Middle Academies, IS218M, in New York City is involved in an integrative investigation to study the history and mathematics of the Inca. The Inca (also spelled Inka) controlled a vast area along the Pacific coast of South America until the Spanish conquered their civilization in the sixteenth century.

Lai and Crespo are two of a growing number of educators who are emphasizing culture in their mathematics instruction. This article describes how they implement these ideas to create an exciting mathematical environment, one in which students not only study important mathematical concepts but also learn to value the mathematical contributions of people who are different from themselves.

**Acquiring Number Sense**

"Imagine that you are in another country. You see some delicious-looking bananas in the market. How can you tell the market vendor that you would like to buy three bananas? You cannot speak the language of the market woman, and she cannot speak yours."

I have often used that introduction with classes of children of various ages, as well as with groups of teachers. The usual reply is, "Point to the bananas and show three with your fingers." I say to the children, "Think how you would show three, then raise your hands and show three on your fingers. Keep your hands up, and look at your neighbors' signs for three. Are they like yours, or different?"

Most raise three fingers on one hand, but a few raise one finger on one hand and two on the other hand. Some bend their fingers rather than raise them. Which fingers do they use? Are the three fingers in consecutive order? Which gestures are the most efficient? Depending on where I am teaching, the gestures are often different, but they seem perfectly appropriate in that setting. The number of "correct" answers amazes students and even some teachers.

How do you show three on your fingers? How would you or your students signify two or four to represent a consistent system of finger gestures? I might repeat the exercise with a larger number, perhaps eight, which would require using two hands. Still, the choices are numerous.

Children acquire number sense through familiarity with numbers. The more often that children work with certain quantities, the easier it is for them to recognize and understand the value of those quantities. The senses transmit the relevant messages to the brain. Of all the senses, the kinesthetic is the most personal, the most directly related to one's own body. Educators recognized the value of tactile experience when they introduced concrete materials into the mathematics class and advocated...
hands-on practice with numbers. What can be more "hands on" than the fingers? Children can actually feel each number as they extend or bend the relevant fingers.

Of course, our other senses are intimately involved in the acquisition of number sense. We hear the sounds of the words assigned to numbers, and we see the symbols that represent various quantities. Numeration systems probably developed in that order: first gestures; then words; and finally, the symbols that we call numerals (Schmandt-Besserat 1999). An investigation into the ways that people in various societies solved the problems of quantification can be a fascinating activity at any grade level (Ifrah 1985). This article describes several examples of numerical systems that illustrate the order suggested. I recommend that you use this order as a format when introducing these systems to your students. You may also want to try an investigation into other numerical systems that are relevant to your locality and its demographics.

Gesture Counting

In many societies, people have developed precise systems of gesture counting that differ from one another just as languages do (Zaslavsky 1999). Gesture counting has a long history, going back to ancient Egypt and China. Some scholars believe that even early humans created gestures to indicate certain quantities. For the merchants of the expanding Roman Empire, finger gestures were the principal mode of communication with their counterparts among the conquered peoples. In the eighth century, the English monk known as the Venerable Bede wrote descriptions of these signs for the numbers up to 9999, illustrating a place-value system based on grouping by tens and powers of ten. These gestures remained in use in the Western world until they were displaced only a few centuries ago by the spread of Hindu-Arabic (also called Indo-Arabic) numerals (Menninger 1992). Students are probably unaware of the role of the fingers in our English number words. For example, eleven and twelve are derived from the words that mean "one left" and "two left" after counting all ten fingers. The Spanish word dedo stands for finger or toe, as does the Latin word digitus from which it is derived. We call the ten symbols for 0 through 9 digits, as in the expression "three-digit number."

The Kamba, the Maasai, and the Taita peoples all live in Kenya in East Africa, yet their finger gestures for the number 8 are quite different. The Kamba hold the middle, ring, and little fingers of the left hand in the right hand (see fig. 1). The Taita extend four fingers of each hand but not the thumb. The Maasai extend four fingers of the right hand and wave them from side to side. In Counting on Your Fingers African Style (Zaslavsky 2000), young children can learn from lively illustrations about the gesture counting practiced by five African ethnic groups in Kenya, Sierra Leone, and South Africa. They are then invited to invent their own systems of finger gestures. The indigenous peoples of the Americas also had their own systems of gesture counting. For example, the Native Americans living on the Great Plains of North America spoke many different languages. Of necessity, they developed a system of mutually understood signs that included gestures for numbers (see fig. 2).

As we shall see, not all societies developed numeration systems based on grouping by tens. Some societies use grouping by twenties, employing all the fingers and toes!

Names for Numbers

Most likely, gestures were the first numerical representations used to communicate quantity. Combining a number name with the gesture followed logically, and in many cultures today, naming a number is still accompanied with a specific hand or finger gesture. Children are fascinated when they explore the names of numbers in various cultures and study the cultural connections of these names. This type of analysis helps us understand how people in these cultures think about counting and numeration.
The Spanish and German languages use number words that are based on groups of ten, whereas French includes some base-twenty words in a system that otherwise groups by tens. Japanese, like Chinese, is a base-ten system, as are most of the languages in use today. However, both the Mende people of Sierra Leone and the Yup’ik Eskimo of Alaska group by twenties and powers of twenty, with subgroups of five and ten. A base-twenty system may seem unusual to children, but a demonstration of the counting principles in base twenty, which are similar to our Hindu–Arabic system, helps children gain number sense. Children who have read Counting on Your Fingers African Style (Zaslavsky 2000) will have encountered the Mende term for 20, nu gboyongo, meaning "a whole person"; in other words, all ten fingers and all ten toes have been counted. A few number words in the Yup’ik language are listed in figure 3 (Lipka, Mohatt, and Ciulistet Group 1998). You might challenge your students to figure out the meaning of akimiaq malruk (17) or yuinaat pingayun (60) using these number words. Yup’ik words for numbers are clearly derived from gesture counting. The word for 5, talliman, means "one arm" (five fingers), whereas qula, the word for 10, means "above," implying that the subsequent number is "below." Indeed, a report published over a hundred years ago indicated a different word for 11, a term that meant "go down" (to the toes). The fact that the word for 20 means "a whole person," twenty fingers and toes, makes sense.

In the base-twenty numeration system of the Yoruba people of Nigeria, a group of five has a special role; it is used for subtraction. For example, the term for 15 means "five from twenty," whereas 35 is expressed as "five from two twenties." This system may seem difficult to us, but it poses no problems for the illiterate market women who, as traders, have traditionally controlled the markets in southwest Nigeria!

Numerical Symbols and Representations

The evolution of numerical symbols and representations signaled the final stage in humankind's efforts to describe quantity. Initially, these invented symbols were pictures drawn of the objects that they represented. Later, other marks that were not pictorially representational were created; these abstract symbols were understood to represent specific quantities regardless of what was being counted. The following paragraphs describe several diverse numeration systems.

Egyptian hieroglyphic numerals

A good place to begin a study of numeration systems is with the five-thousand-year-old Egyptian system of hieroglyphics. This system uses a different symbol for each power of ten and repeats the symbol as many times as necessary; therefore, a symbol for zero is unnecessary. The fact that the Egyptians invented one of the very first numeration systems with no models to follow is mind boggling. Why did they adopt ten as a base? No doubt the answer is because of the ten fingers. Egyptian numerals were usually written with the smallest value on the left. Symbols were grouped for convenience, although the number can still be read correctly even when the symbols are mixed up. For example, 2 013 246 would be written as shown in figure 4, with the symbols in the correct order.

Chinese rod numerals

Chinese rod numerals, also called stick numerals, are based on a system that had its origin over two thousand years ago and can probably be traced back to finger gestures. The system is based on grouping by powers of ten with subgroups of five. The Chinese abacus has similar groupings. Bamboo sticks are laid out on a table in columns headed one, ten, hundred, thousand, and so on, a positional system similar to our Hindu-Arabic system. An empty column represents our zero. The sticks are laid out mainly in either a horizontal or a vertical direction, depending on the column in which they appear. The directions alternate from one column to the next. Figure 5 shows several representative numerals.
Mayan base-twenty numerals
The Maya of southern Mexico and northern Central America use a base-twenty, subbase-five system that is over two millennia old. The symbols are bars and dots, which were no doubt derived from marketplace calculations using twigs and pebbles. In this place-value system, the lowest value is recorded at the bottom of the vertical column. The Maya probably had one of the first written symbols for zero as a placeholder in a written numeral. Figure 6 shows some typical Mayan numerals, found on their stone monuments and in written documents dealing with astronomy and other learned subjects.

Inca Quipu
The quipu is a collection of different-colored cotton or wool strings or cords used by the ancient Inca to count and record a variety of data that were important to daily life and society, including census data, contents of storehouses, and output of gold mines (Ascher and Ascher 1981, 1977). On each cord, knots indicate various quantities in a base-ten place-value system. A separate string shows the total of a group of cords. An empty space in any position represents zero (see fig. 7).

Hindu-Arabic system
The Hindu-Arabic numeration system is considered one of the most important inventions in the history of humankind. With some variations, it has spread throughout the world, an instance of globalization that benefits all. The beauty of this system lies in the following three features: (1) separate symbols for the numbers from 1 through 9, (2) a symbol for zero, and (3) place value for successive powers of the base number 10. The system originated in India in about the seventh century and was spread by Arabic-speaking merchants and scientists, hence the name "Hindu-Arabic" or "Indo--Arabic." Only after several more centuries elapsed did the system become standard in Europe. Of course, children study and use this system today. The following section examines how students in several classrooms acquired a deeper awareness of the Hindu-Arabic system by actively exploring systems of other cultures.

Classroom Investigations
As described previously, Bianka Crespo and her fifth-grade students studied the mathematics of the Inca. They became aware that the knots of the quipu indicated quantities in a base-ten place-value system. Students were challenged to discover that the top cord showed the total of a group of cords and that an empty space in any position represented zero in that position. As a culminating activity, each group of students devised a list of the quantities of potatoes required by each family in the village. They then tied knots that were carefully spaced apart on various colors of yarn, creating the required numbers of hundreds, tens, and ones to be read on their Inca quipus.

Students were fascinated by the mathematics of the quipu. One exercise required that they mentally add the knots on the three lower cords and record their results on the top cord. One student described her unique solution. She rotated her paper 90 degrees counterclockwise so that the units were in a column on the right. She then added in the usual manner, starting with the units place and regrouping as necessary (see fig. 8).

To emphasize the process of regrouping in addition, Crespo asked her students whether it was possible to attain the correct sum if they started by adding the knots in the hundreds place rather than those in the units place. After working several examples, students concluded that the result was the same, no matter in which place one started the process. One boy challenged the other students by asking whether they could add the knots in the three cords along a diagonal and obtain the correct result! Several students explained in their own words that each set of knots referred to a different power of ten and used the analogy of adding a similar set of Hindu-Arabic numerals across the diagonal.
Positional notation became meaningful to this class. As Crespo commented, "Lessons with the quipu are perfect for reinforcing students' understanding of place value" (Zaslavsky, forthcoming). In an illustrated essay explaining the mathematics of the quipu, one student, Maribel, wrote, "A quipu is a calendar to keep track of important dates in your daily life. It keeps you organized."

Linda Lai used her growing understanding of the importance of connecting mathematics with culture by creating a unit entitled "Everybody Counts." Her mathematics instruction focused on the diverse counting methods used by different cultures and on their systems of numeration. She wanted her students to deepen their understanding of number sense by studying multiple representations of counting practices from a variety of cultures. (See Zaslavsky [1996] for more information on multicultural mathematics activities.)

Students manipulated black Cuisenaire rods instead of bamboo sticks to form Chinese rod numerals. A Chinese abacus was available at the mathematics center to allow students to explore addition and subtraction using beads. Lists of Alaskan Yup'ik, Nigerian Yoruba, and Japanese numbers, along with Plains Indian number gestures, were posted, and their patterns were discussed. Ancient Mayan numbers were explored using counters, Cuisenaire rods, and construction-paper ellipses; and students experimented with drawing the glyphs of the Egyptian hieroglyphic system.

The students were intrigued by, and motivated to learn about, the variety of systems that people have invented for counting and keeping records. Linda Lai had the following conversation with a student concerning his interpretation of Mayan numerals using Cuisenaire rods and counters, an excellent tactile experience.

Lai OK, how would you make 18? [Matt.placed eight counters over two rods, thus forming the equation 8 + (2 x 5).]
Lai. Can you think of another way to make this number using this system?
Matt. Well, you could put another 5 rod and leave three counters [3 + (3 x 5)].

Matt had discovered two correct answers to the question. Of course, the Maya had chosen the most efficient method of representing the number 18 in their system, using three bars and three dots. Lai reminded her students that this system was used long ago and asked them whether they had seen a similar system in use today. After a bit of prodding, the students eventually discovered that bar codes on grocery labels use a counting system interestingly similar to that of the Maya (Zaslavsky 1996).

After this exploration, Lai wanted to see how much her students had learned by challenging them to invent their own numeration systems. First, the children decided on the quantities for which to create symbols. To help the children organize their work, Lai distributed sheets, organized around a base-ten system, on which students could record their invented symbols. Nick invented "kumbers" (see fig. 9).

Nick said that he needed a unique name for his system because it was going to be different and "it rhymed with numbers." He explained his drawings by saying that as the values increased, he wanted his symbols to become more complex. Although most of his symbols were drawn simply because they "looked neat," the symbol for 1000 was purposely drawn to represent the sun because 1000 of anything is a lot, and as Nick explained, "the sun lasts a long time." Nick said that inventing his own system was fun and easy. He learned that people who made the other numeration systems that he and his classmates studied "must have been very smart to come up with all of those numbers in such different groups."

Lai commented, "This activity turned into an extremely valuable assessment tool on our numeration unit as I conferenced with each student about their numerals and their meaning." The students addressed issues of place value, order of numbers from least to greatest or greatest to least, and
methods for writing numbers most efficiently. Lai added that these activities connect well with the messages of the NCTM's Standards documents (1989, 2000) and help her students see that more than one approach to a problem is possible. She thought that her students related well to the multicultural mathematics instruction and mused, "In my classroom, we spend a lot of time looking for similarities among people regardless of the apparent differences... everyone has so much in common... everyone has math!"

Conclusion
Crespo and Lai are enthusiastic about the mathematical learning that has taken place in their classrooms as they seek additional opportunities to connect mathematics and culture. Students can begin to appreciate the Hindu-Arabic numeration system better as they learn about and experience the many ways that people have counted and recorded numbers throughout the development of society over a period of thousands of years. Students not only gain in their understanding of other mathematical systems but also develop greater appreciation of the fact that all people count.

Students learn that numerical systems can be very different yet possess many strikingly similar, vital mathematical principles. Although differences can set people apart, children can be guided to see the commonalties in our lives. What a fascinating history of humankind is portrayed through numbers!

[Reference]
References

[Reference]


[Author note]
Claudia Zaslavsky is retired after many years of teaching secondary-level mathematics and teacher education classes. She is the author of numerous books related to multicultural perspectives in mathematics.

Reproduced with permission of the copyright owner. Further reproduction or distribution is prohibited without permission. &Basic &Advanced &Guided &Publication &Natural language &Last search &Recent searches &Marked list &durable links